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X.

CONTRIBUTIONS FROM THE PHYSICAL LABORATORY OF THE MASSACHUSETTS INSTITUTE OF TECHNOLOGY.

XXVII. — THE INVERSE ELECTROMOTIVE FORCE OF THE VOLTAIC ARC.

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THE fact that the total or equivalent resistance of a voltaic arc consists of two parts, the one constant for the same current, the other variable in proportion to the length of the arc, was first made known by Edlund in 1867 and 1868.* This total resistance, r, is evidently represented by the equation $r = a + bl$, and if the constant a be multiplied by the current, C, employed, the inverse electromotive force is found.

Later researches have shown conclusively, with the silent arc, throughout a great range of length and of current strength, that this product, aC, is approximately constant.

The most recent results on this subject are those of Fröhlich,† whose results give a value of 39 volts for the quantity aC; of Peukert,‡ who finds a mean value of 35 volts for currents of 10 ampères and over; and Von Lang,§ who assigns a value of 39 volts.

It has long been known that on the passage of the arc from the whistling to the silent stage, as the carbons are separated, there is a sudden rise in the difference of potential between them. At the same time, the arc becomes much brighter and hotter, from which we should naturally infer that the conductive resistance would diminish. The current, however, is at the same time diminished.

* Poggendorff's Annalen, 1867, vol. cxxxi. p. 586; 1868, vol. cxxxiv. p. 250.

† Elektrotechnische Zeitschrift, 1883, vol. iv. p. 150.

‡ Zeitschrift für Elektrotechnik, 1885, vol. iii. p. 111.

§ Centralblatt für Elektrotechnik, 1885, vol. vii. pp. 315, 446.

These facts might lead one to infer the probability of there being a sudden rise in the inverse electromotive force of the arc at the point when whistling ceases. So far as we are aware, this matter had not been studied previous to the observations which we have to present.

The quantity aC was originally supposed by Edlund to be of the nature of an opposing electromotive force. It is true that this view has not been accepted without question, and that the real cause of the phenomena under consideration is still somewhat uncertain; but we shall, nevertheless, in this paper, for convenience of description, refer to this quantity aC as the inverse electromotive force of the arc.

The object of the experiments described in the present paper was to review the results of earlier observers on the quiet arc, to extend the observations to the whistling arc, and to observe the effect on the inverse electromotive force of the arc caused by variations in the position and temperature of the electrodes, by the presence of volatilized substances in the arc, and by variations in the density of the air in which the arc is formed.

A lamp was constructed for the purpose whose upper carbon was movable in a vertical direction by a micrometer screw, having a pitch of $\frac{1}{32}$ of an inch. Wires ran from brass collars surrounding the carbons, and as near as possible to their extremities, to a voltmeter, which gave the difference of potential between the carbons. The current of electricity was usually furnished by a Brush dynamo machine. It was measured in some cases by means of a carefully and frequently calibrated Ayrton and Perry permanent magnet ammeter, and in others by a Paterson and Cooper electromagnet ammeter, also calibrated by comparison with standard instruments. A similarly calibrated Ayrton and Perry permanent magnet voltmeter was employed for the potential measurements. Reversals of readings were obtained by a mercury commutator so arranged as to reverse both the current and potential instruments simultaneously. The quotient of the difference of potential between the carbons divided by the current gave the equivalent resistance of the arc. Care was taken that the arc should be formed so as to pass between those points of the carbons which were closest together, which was secured by filing them flat. The separation between the points as measured by the micrometer, therefore, gave the length of the arc. It was of course necessary to make the measurements with a sufficient degree of quickness, so that no sensible burning away should occur before they were finished. No difficulty was found in doing this, as it took a number of seconds

for the filed carbons to shorten by any material amount. Before each set of measurements the carbons after filing were screwed into contact, the reading of the micrometer taken, and the carbons were then separated by the desired amount. It was found, however, that the expansion of the carbons by heat was a noticeable quantity, and hence, except with very long arcs, the arc was allowed to form, and then the carbons were quickly screwed into contact and again separated by the desired amount. For arcs so long that this was not practicable, the expansion was estimated and allowed for. The change of reading from this cause appeared to vary from $\frac{1}{20}$ to $\frac{1}{4}$ of a revolution of the micrometer screw; that is, from $\frac{1}{640}$ to $\frac{1}{128}$ of an inch. In order to avoid undue disturbance of the dynamo during the measurements, the dynamo circuit was never broken, but by means of a suitable key the current was thrown into a wire resistance approximately equivalent to the arc resistance before the arc was broken. The current forming the arc was regulated by means of suitable resistance coils interposed in the circuit.

In most of the measurements Boulton carbons were used. Ten successive measurements with reversals were usually made in each series, keeping the current and electromotive force as nearly constant as possible, and taking the mean value as the result of one set of measurements for the length of arc used. The two following tables, taken at random, are given in full to indicate the degree of variation observed. One of the tables shows results with a silent arc, one with a whistling arc. The column headed A contains the scale readings of the ammeter; that headed V, those of the voltmeter. The mean of each of these is taken, and the current and electromotive force desired are then found by multiplication of the mean by the proper instrumental constants, which, with the tables given, are 1.00 for the ammeter and 2.12 for the voltmeter.

TABLE I. — Silent Arc.

Length of Arc = $\frac{3}{32}$ in.

A.	V.
5.3	21.2
5.3	21.5
4.9	22.2
5.1	21.0
5.1	21.7
5.1	21.2
4.8	22.7
5.0	21.2
4.9	22.0
4.8	22.2
5.1	22.2
5.2	21.8
5.1	22.2
5.1	22.2
5.1	23.0
5.2	22.8
4.8	23.8
4.9	23.7
4.8	24.2
4.5	24.5
Mean, 5.0	22.37

C = 5.00 ampères.
E = 47.42 volts.
R = 9.48 ohms.

TABLE II. — Whistling Arc.

Length of Arc = $\frac{3}{32}$ in.

A.	V.
5.2	19.0
5.3	18.2
5.1	18.2
5.1	18.2
5.2	18.2
5.2	18 2
5.3	18.0
5.1	19.0
4.9	19.0
4.8	19.8
4.8	19.0
4.8	19.0
5.1	18.8
5.2	18.2
5.1	18.5
4.9	19.2
4.9	18.8
5.0	18.2
5.0	19.0
5.0	19.0
Mean, 5.05	18.67

C = 5.05 ampères.
E = 39.59 volts.
R = 7.84 ohms.

Owing to the great variability of the arc, it is of course impossible to avoid having a considerable average deviation with a series of successive measurements of current and electromotive force; but, as will be seen, in spite of the necessarily large limits of variation, consistent results are nevertheless obtainable.

About 1200 measurements were taken, embracing 60 series of 10 readings with reversals, using lengths of arc of from $\frac{1}{128}$ in. to $\frac{1}{2}$ in., and currents varying from 3 to 10 ampères. Ordinarily we made a series of measurements with increasing lengths of arc, keeping the current as nearly constant as possible by means of resistance coils. The variations in the current were rarely more than a few hundredths of an ampère, and as the current was measured simultaneously with the electromotive force, these small changes did no harm. The equivalent resistance for each length of arc, with the various strengths of current, was obtained from the data given by the measurements. The following tables show the results reached. The first column gives the

separation of the carbons, which is the length of the arc, the unit being $\frac{1}{32}$ of an inch; the second column gives the equivalent resistance in ohms, which is the electromotive force between the carbons divided by the corresponding value of the current; the third column states whether the arc was a silent (S.) or whistling (W.) one. The value of the current given over each table is the mean of the values found in the various separate measurements.

UPRIGHT ARC.

TABLE III.

Current = 3.27 ampères.

Length of Arc.	Resistance.	Character.
0.25	5.51–5.93	W.
0.50	7.41	"
0.50	6.96*	"
1.00	9.50	"
1.00	9.86*	W. (slightly.)

TABLE IV.

Current = 5.04 ampères.

Length of Arc.	Resistance.	Character.
1	5.67	W. (slightly.)
2	8.14	S.
3	9.51	"
4	10.02	"
5	10.57	"
6	11.10	"
7	11.60	"

TABLE V.

Current = 7 ampères.

Length of Arc.	Resistance.	Character.
0.25	2.49	W.
0.50	2.89	"
1.00	3.60	"
1.00	3.55	"
2.00	5.28	"
2.00	5.17	" (low.)
3.00	6.28	S.
3.00	6.41	"
4.00	6.77	"
4.00	6.72	"
5.00	6.97	"
6.00	7.25	"
7.00	7.51	"
8.00	7.80	"
9.00	8.06	"
12.00	8.97	"
12.00	8.90	"
14.00	9.31	S. (flaming.)

TABLE VI.

Current = 7.94 ampères.

Length of Arc.	Resistance.	Character.
0.50	2.42	W.
1.00	3.09	"
2.00	4.58	W. (slightly.)
3.00	5.60	S.
4.00	6.04	"
5.00	6.32	"
6.00	6.49	"
7.00	6.73	"
8.00	7.14	"
10.00	7.51	"
12.00	8.09	"
14.00	8.57	"
16.00	8.89	"

* These results were given by Carré carbons.

TABLE VII.

Current = 10.04 ampères.

Length of Arc.	Resistance.	Character.
0.50	1.86	W.
1.00	2.30	"
1.00	2.19	"
1.00	2.34	"
2.00	2.95	S.
3.00	3.94	"
3.00	3.74	W. (low.)
4.00	4.45	S.
5.00	4.67	"
6.00	4.91	"
7.00	5.08	"
8.00	5.32	"
8.00	5.22	"
10.00	5.66	S. (flaming.)

From the measurements as given in the tables, curves were next plotted on a large scale for each mean strength of current, with the separations of the carbons as abscissæ and the corresponding resistances as ordinates.

These curves were in all cases found to be composed of two straight lines, the intersection of which corresponds approximately to the point at which the arc, as it is gradually lengthened, passes from a whistling to a silent one. The curve for the whistling arc is the steepest. Hence, for the curve corresponding to each strength of current there are two different intercepts and two different angles made with the axis of abscissæ, according as the arc is whistling or silent. Since the intercept gives the quantity a in the general equation $r = a + bl$, which quantity being multiplied by the current gives the corresponding value of the inverse electromotive force of the arc, it follows that for any particular current there is a definite opposing electromotive force for the whistling arc as for the silent arc, whatever its length, but that the former is different from and less than the latter. Also, there is for the whistling arc, as well as for the silent arc with any definite current, a definite value of b, and hence a simple proportionality between the variable portion of the equivalent resistance and the length. The values of b are greater with the whistling than with the silent arc for the same current strength.

From the curves plotted as described, the linear equations of the two branches corresponding to the silent and whistling arcs were

obtained. These are given in Tables VIII. and IX., and are numbered for convenience of reference. The length, l, is expressed in terms of the pitch of the micrometer screw, i. e. in thirty-seconds of an inch. The current is expressed in ampères. Lengths of arc up to $\frac{16}{32}$ in. were observed.

TABLE VIII. — SILENT ARC.

Current.	Number.	Equation.	E. M. F. $= a \times C$.
5.04	1	$r = 7.97 + .515\,l$	40.16
7.00	2	$r = 5.73 + .261\,l$	40.11
7.92	3	$r = 5.00 + .256\,l$	39.60
10.04	4	$r = 3.73 + .198\,l$	37.45
			Mean, 39,33

TABLE IX. — WHISTLING ARC.

Current.	Number.	Equation.	E. M. F. $= a \times C$.
3.27	5	$r = 4.70 + 4.90\,l$	15.37
5.03	6	$r = 3.14 + 2.53\,l$	15.79
7.00	7	$r = 2.10 + 1.55\,l$	14.70
7.95	8	$r = 1.81 + 1.26\,l$	14.39
10.03	9	$r = 1.46 + 0.787\,l$	14.64
			Mean, 14.98

The fourth column of the table gives the inverse electromotive force corresponding to each current, as found by multiplying the intercept given in the equation by that current. It appears from the results, that for the silent arc the mean value of the inverse electromotive force is approximately 39 volts. There is, however, an evident diminution in this value as the current rises, and by reference to the equations given in Peukert's paper, already cited, it will be seen that they also indicate the probability of such a change.

When the arc becomes very long there is an apparent tendency to an abnormally small value of the equivalent resistance. This was observed in the values of the resistance when the arc was $\frac{14}{32}$ in. long, with a current of 7 ampères, and when the arc was $\frac{16}{32}$ in. long with a current of 8 ampères. It was also noticed with very long arcs in which metallic salts were volatilized.

Table IX. also shows us an additional fact. Not only is the inverse electromotive force of the silent arc the same for any given current,

irrespective of the length of the arc (within the limits of whistling), but it also has, like that of the silent arc, an approximately definite value for all currents. The mean of the results with the silent arc given in the table is about 15 volts. There is, also, notwithstanding some anomalous results, the same manifest diminution with increase of current that we observe with the silent arc. It will further be noticed, that the constant, b, diminishes as the current increases, but that its value is several times as great for the whistling as for the silent arc.

For both the silent and whistling arcs, the conductive resistance, bl, diminishes as the current is increased, at first rapidly, afterwards more slowly.

The equations verify previous observations, in that the passage from higher to lower inverse electromotive force is a sudden one. It takes place with a greater separation of the carbons, and less equivalent resistance, as the current is stronger, a result agreeing with that reached by White from direct measurements.* The following figures (Table X.), taken from the curves as plotted, illustrate this. The first column gives the approximate current, the second the equivalent resistance, the third the separation of the carbons at which the change would take place, which last is, of course, the abscissa of the point of intersection of the two straight lines representing the two varieties of arc.

TABLE X.

Current.	Equivalent Resistance.	Distance between Carbons.
5.0	9.22	$\frac{2.4}{32}$ = .075 in.
7.0	6.46	$\frac{2.8}{32}$ = .087 in.
7.9	5.83	$\frac{3.2}{32}$ = .100 in.
10.0	4.46	$\frac{3.8}{32}$ = .119 in.

These values would doubtlessly vary considerably with the nature of the carbons used for the electrodes.

It was also noticed with the stronger currents, that, on lengthening the arc, the whistle ceased slightly before the higher inverse electromotive force was reached. This appeared to be somewhat influenced by the quality of the carbons. In all cases, however, the line of demarcation between the silent and the whistling arc was very defi-

* Electrician, 1884, vol. xiv. p. 56.

nite. With the whistling arc, there was always great unsteadiness, which was very much diminished at the moment when the arc became quiet. At the same time the positive carbon suddenly brightened, indicating a great rise of temperature.

A quite interesting peculiarity was noticed in the curves for the whistling arcs. As will also be seen from the equations, the lines all converge approximately towards a point situated behind the axis of ordinates, and corresponding algebraically to a negative separation of about $9 \times \frac{1}{32}$ inches, the corresponding algebraic value of the resistance being about $\frac{7}{10}$ ohms. This property of the curves furnishes a means of finding approximately the equivalent resistance of any whistling arc, as follows. Divide the average value of the inverse electromotive force, 15.13 volts, by the strength of the current, which gives the intercept, a, corresponding to the current in question. Draw a line from the point of convergence as given above, through the upper point of the intercept, and it will be the curve for this current. The ordinate corresponding to any abscissa gives the equivalent resistance for that length of arc. The curve for the smallest current used, 3.27 ampères, alone shows much deviation from the rule. The curves for the silent arcs do not show any such definite point of convergence.

It is interesting to examine Edlund's early values for the inverse electromotive force in the light of the preceding results. His first values are expressed in units which are purely arbitrary. They show approximately, however, the same figure for $a\,C$ through a considerable range of variation in the strength of the current. His later results give 9.7 as the value of the inverse electromotive force with a current of 30 Bunsen elements, the electromotive force of a Bunsen cell being taken as a unit. With 50 elements its value rose to 15. It seems probable that the former result was reached with a whistling arc, and the latter with a silent one. It is difficult to explain the existence of certain intermediate values, but the method by which they were obtained is liable to great errors.

If we consider Peukert's paper, we find that the results from which he derives his value of 35 do not include any with a less current than 10 ampères, and but one in which the length of the arc is less than 2 mm., so that the arcs were probably all silent. The observations from which Fröhlich constructs the curve giving the value of 39 are too irregular to show any difference that might exist between the values of the constant for the silent and for the whistling arcs. Von Lang states that the arcs measured by him were carefully kept from hissing by manual adjustment.

A large number of measurements were made with an inverted arc, in which the positive carbon was below and the negative one above.

Currents of approximately 5, 7, and 10 ampères were used, with lengths of arc up to $\frac{7}{32}$ inch. There was much unsteadiness in the voltmeter for arcs more than $\frac{3}{32}$ inch long, there being a slow and irregular fluctuation of the needle between definite limits. Very probably this was due to irregularities in the air currents rising from the lower and hotter positive carbon. In some sets of the measurements (Series II. of tables) the extreme readings were taken and separate curves drawn for each. The following tables (XI. to XVIII.) give the results of the measurements.

Inverted Arc. — Series I.

TABLE XI.

Current = 5.1 ampères.

Length.	Resistance.
0.50	3.88
1.00	4.91
2.00	7.50
3.00	9.00
4.00	9.50
5.00	9.90
6.00	10.60

TABLE XII.

Current = 7.3 ampères.

Length.	Resistance.
1	3.22
2	4.64
3	5.85
4	6.49
5	6.76
6	6.91
7	7.14
8	7.38

Inverted Arc. — Series II.

TABLE XIII.

Current = 5.1 ampères.

Separation.	Resistance.	
	Minimum.	Maximum.
0.50	3.86	....
1.00	5.05	5.48
2.00	6.98	8.23
3.00	8.81	9.52
4.00	9.33	10.38
4.00	9.14	9.97
5.00	9.52	10.60
6.00	9.81	11.01
7.00	10.40	11.43
8.00	10.40	12.00
9.00	10.80	12.50

TABLE XIV.

Current = 10.2 ampères.

Separation.	Resistance.	
	Minimum.	Maximum.
3	3.06	3.83
4	3.60	4.70

TABLE XV.

Current = 9.6 ampères.

Separation.	Minimum.	Maximum.
5	4.10	5.17
6	4.18	5.27
7	4.18	5.60

The curves for the inverted arc were constructed, as already explained for the upright arc. These were found, as with the latter, to consist of two distinct straight lines, corresponding to the silent and the whistling arcs respectively. The following equations (Tables XVI., XVII.) represent the results obtained. Equations 10 and 12 were found by taking the extreme readings as described, the current being steady at 5.1 ampères. Equation 11 was obtained from another series of experiments, using the same strength of current, but taking the mean reading of the voltmeter at each observation. Equations 14 and 15 were constructed from the extreme readings. The current varied slightly with these last, the average value being 9.6 ampères for the silent arc and 10.2 ampères for the whistling arc. The inverse electromotive force is given in the last column of the table.

TABLE XVI. — Silent Arc.

Current.	Number.	Equation.	E. M. F. $= a \times C.$
5.1 (max.)	10	$r = 7.70 + .540\ l$	39.27
5.1	11	$r = 7.70 + .450\ l$	39.27 } 39.27
5.1 (min.)	12	$r = 7.70 + .367\ l$	39.27
7.3	13	$r = 5.60 + .214\ l$	40.88 - 40.88
9.6 (max.)	14	$r = 4.01 + .230\ l$	38.50 } 38.07
9.6 (min.)	15	$r = 3.92 + .052\ l$	37.63
			Mean, 39.41

TABLE XVII. — Whistling Arc.

Current.	Number.	Equation.	E. M. F. $= a \times C.$
5.1 (max.)	16	$r = 2.95 + 2.67\ l$	15.04
5.1	17	$r = 2.85 + 2.07\ l$	14.54 } 14.71
5.1 (min.)	18	$r = 2.85 + 2.03\ l$	14.54
7.3	19	$r = 1.95 + 1.32\ l$	14.23 - 14.23
10.2 (max.)	20	$r = 1.30 + 0.846\ l$	13.26 } 13.26
10.2 (min.)	21	$r = 1.30 + 0.583\ l$	13.26
			Mean, 14.07

It will be seen from the figures in the last column that the fluctuations of the voltmeter to which we have referred were, for the most part, due to variations in the conductive resistance of the arc, the inverse electromotive force being almost constant for the same current, notwithstanding the variations occurring in the total resistance. With the whistling arc the inverse electromotive force is decidedly

less for the same current in the inverted than in the upright arc, and the same is probably true to a less degree with the silent arc, though this is not so clearly shown in our observations, partly on account of an exceptionally high value for the inverse electromotive force with the current of 7.3 ampères and inverted arc.

It will also be observed that the corresponding values of the conductive resistance (bl) and the total equivalent resistance are both somewhat less for the inverted arc. This differs from a result somewhat doubtfully recorded by Niaudet.*

The points of intersection of the lines corresponding to the two varieties of the inverted arc are given in the following table, as read from the curves. They may also, of course, be obtained by solution of the equations.

TABLE XVIII.

Current.	Equivalent Resistance.	Distance between Carbons.
5.1 (max.)	8.92	$\frac{2.2}{32}$ = .069 in.
5.1	9.05	$\frac{3.0}{32}$ = .094 in.
5.1 (min.)	8.74	$\frac{2.9}{32}$ = .091 in.
7.3	6.32	$\frac{3.3}{32}$ = .103 in.
9.6–10.2 (max.)	5.02	$\frac{4.4}{32}$ = .137 in.
9.6–10.2 (min.)	4.16	$\frac{4.9}{32}$ = .153 in.

Comparing this with the table already given for the upright arc, it appears that with the inverted arc the length at which the higher inverse electromotive force sets in is somewhat greater. The corresponding equivalent resistance is not greatly different in the two cases.

Measurements were also made of the resistance of arcs into which certain metallic salts had been introduced. The salt, in powder, was placed on the lower (negative) carbon while this was quite hot, care being taken entirely to cover the carbon point, and to keep the powder from being jarred off on forming the arc. The reading of the voltmeter for a given length of arc remained quite constant for a short time after the arc was established, so that readings could readily be obtained.

Observations were made with arcs containing biborate of soda, sulphate of soda, and sulphate of potash. The arcs employed were from $\frac{1}{32}$ to $\frac{4}{32}$ in. in length with the biborate of soda, from $\frac{1}{32}$ to $\frac{5}{32}$ with the sulphate of soda, and from $\frac{1}{32}$ to $\frac{16}{32}$ with the sulphate of

* Machines Electriques à Courants Continus, p. 133.

potash. The current was approximately 5 ampères. The equation $r = a + bl$ still expressed the results as follows: —

TABLE XIX.

Current.	Salt.	Number.	Equation.
5.10	Biborate of Soda	22	$r = 1.25 + .290\, l$
4.94	Sulphate of Soda	23	$r = 2.20 + .418\, l$
5.10	Sulphate of Potash	24	$r = 1.77 + .223\, l$

The values of the inverse electromotive force were for biborate of soda 6.38 volts, for sulphate of soda 10.86 volts, and for sulphate of potash 9.03 volts. The conductive resistance, as well as the inverse electromotive force, is diminished by the presence of the salts in the arc.

The data from which the curves giving the preceding equations were constructed, are given in the following tables.

TABLE XX.

BIBORATE OF SODA.

Current = 5.10 ampères.

Length.	Resistance.
1	1.53
2	1.82
3	2.12
4	2.41

TABLE XXI.

SULPHATE OF SODA.

Cnrrent = 4.94 ampères.

Length.	Resistance.
1	2.64
2	2.98
3	3.43
4	3.85
5	4.00

TABLE XXII. — SULPHATE OF POTASH.

Current = 5.10 ampères.

Length.	Resistance.
1	2.00
2	2.21
3	2.49
4	2.71
5	2.96
6	3.08
7	3.29
9	3.82
11	4.21
13	4.58
15	4.91

Whatever may prove to be the real cause of the inverse electromotive force of the arc, it is clear that it is intimately connected with variations in the temperature of the carbons. Thus both the actual temperature and the difference of temperature between the carbons is greater with the silent than with the whistling arc, and at the same time the inverse electromotive force of the former is the higher. Again, the temperature of the carbons is apparently lowered when metallic salts are introduced into the arc, and the inverse electromotive force falls at the same time. Edlund * found that by heating the negative carbon by a blast-lamp, the inverse electromotive force was raised; but this was not the case when the flame of the lamp was directed against the positive carbon, and in fact there then seemed to be a fall. It seems very probable that this last result came from the fact that the positive carbon was so much hotter than the flame that the effect of the latter was on the whole to cool the arc. We endeavored to increase the temperature of the positive carbon in a different manner, by surrounding it with a deep cup-shaped shield of fire-clay, which projected over the end of the carbon so as to envelop the arc, thus preventing radiation. At the same time a broad, flat, horizontal plate of brass was attached to the lower and negative carbon. This obstructed the access of air to the arc, and thus tended to keep up its temperature.

A current of 7 ampères was employed, with arcs varying from $\frac{1}{32}$ to $\frac{12}{32}$ in. in length. As the carbons could not be filed in these experiments, the length of the arc was obtained by screwing the points into contact while the current was passing, and then separating them to the required distance. A moderately high wire resistance was placed in the circuit with the arc, so that there was no material change in the strength of the current and condition of the dynamo machine caused by this adjustment of the carbons, and the needed measurements could be made rapidly. There was considerable periodic fluctuation of the reading of the voltmeter, as had been noticed previously with the inverted arc, and two curves were drawn corresponding to the extreme readings. The current was sensibly constant. The highest set of values for the equivalent resistances corresponding to different lengths of arc are satisfied for the silent arc by the equation (25) $r = 6.70 + .16\, l$, and for the whistling arc by the equation (26) $r = 3.02 + 1.10\, l$. The corresponding values of the inverse electromotive force are 46.9 volts and 21.1 volts. The lowest set

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* Pogg. Ann., vol. cxxxiv. p. 250.

of values are satisfied by the equations (27) $r = 6.70 + .09\, l$ for the silent arc, and (28) $r = 3.02 + .97\, l$ for the whistling arc. The corresponding values of the inverse electromotive force are the same as before, viz. 46.9 and 21.1, respectively, showing that the changes observed in the difference of potential between the carbons were due to variations in the conductive resistance, and not to variations in the inverse electromotive force of the arc. These resistance variations, while amply large to affect the readings of the voltmeter, were too small to produce any sensible change in the strength of the main current. It also appears by a comparison of the equations just given for the shielded arc with equations (2) and (7) that the coefficient of l is decidedly less with the shielded arc, showing that the conductive resistance of the arc is diminished by the increased temperature, as would be anticipated. The increased inverse electromotive force is much greater than we had expected to find it, and at first might lead one to surmise why, with the ordinary arc, the inverse electromotive force does not rise with increased strength of current. This is probably to be explained by the fact that the chief effect of an increase in current is to heat a larger area of the carbons to the same temperature, rather than to raise them to a considerably higher temperature.

Observing the position of the point of intersection of the lines representing the silent and whistling arcs, we find that the passage from low to high inverse electromotive force occurs with a length of arc of $\frac{3.92}{32} = .123$ in. for the line represented by equation (26), and of $\frac{4.20}{32} = .131$ in. for that of equation (28), which values are greater than those occurring with the normal arc and the same strength of current. The corresponding equivalent resistances are 7.34 ohms and 7.07 ohms.

We next tried the reverse experiment of cooling the upper positive carbon. This was done by surrounding the lower end of the carbon with a brass tube through which a current of cold water flowed. The negative electrode was left in its usual condition. The arc under these conditions was extremely unstable, the slightest breath would extinguish it, and even with a current of 10 ampères it could not be elongated to a greater length than $\frac{3}{32}$ in. It was quite blue, whistled, and gave but very little light. The extreme length of arc used was $\frac{3}{32}$ in. with the weaker, and $\frac{2}{32}$ in. with the stronger current. Plotted curves gave the following equations, satisfying the observations with tolerable exactness.

Current.	Number.	Equation.
7	29	$r = 1.67 + 1.11\, l$
8	30	$r = 0.70 + 1.82\, l$

The corresponding inverse electromotive forces are 11.7 volts and 5.6 volts respectively. Both of these are much below the value already found for the whistling arc under normal conditions. But we also find here, for the first time, a great difference between the inverse electromotive force with different currents. As with the normal arc, the higher value occurs with the lesser current. With the current of 7 ampères the conductive resistance is lower than with the normal arc, but with 8 ampères it is higher. Owing to the low inverse electromotive force, the total resistance is considerably less with both strengths of current.

It was our intention to observe also the effect of cooling the negative carbon, but this was prevented by an accident to the apparatus. There is no reason to suppose, in the light of Edlund's experiments already cited, that this would have made any change in the nature of the results.

This fall of inverse electromotive force with lowered temperature of the arc may be the explanation of the fall which occurs with the inverted arc, as the increased convection when the hot positive crater is turned upward may be sufficient to lower the temperature by a slight amount.

The following tables give the data from which our equations were determined.

TABLE XXIII.

Positive Carbon Heated.

Current = 7 ampères.

Length.	Resistance.	
	Minimum.	Maximum.
1	3.86	4.24
2	4.91	5.15
3	5.90	6.36
4	6.97	7.42
5	7.42	7.42
6	7.27	7.66
7	7.31	7.88
8	7.21	7.97
9	7.57	8.10
10	....	8.48
11	....	8.64
12	....	9.10

TABLE XXIV.

Positive Carbon Cooled.

Current = 7 ampères.

Length.	Resistance.
1	2.72
2	3.96
3	4.99
Current = 8 ampères.	
1	2.52
2	4.35

For observations on the arc formed under diminished pressure and in different gases, a metallic receiver was constructed in which the rod carrying the upper carbon passed through the closed top of the receiver, and was movable by means of a micrometer screw. A win-

dow set in the side of the receiver allowed observation of the arc. The exhaustion could be read by an attached gauge. The rubber screw plug, through which the rod passed, was covered with oil or glycerine to diminish the leakage. The construction of the apparatus was greatly delayed by unforeseen circumstances, so that up to the present date we have been able to make only a very few experiments. Some difficulty has also been met with from the heating and vaporization of portions of liquid which have leaked into the receiver. This was partially avoided by immersing the receiver in water. In the experiments thus far made with this apparatus, the pressure was 4 inches of mercury. Currents were employed of 5, 7.95, and 9.86 ampères respectively, with lengths of arc from $\frac{1}{64}$ to $\frac{6}{32}$ inch. In all the experiments the positive carbon was uppermost. The arc was found to be a whistling one up to the extreme limits obtained. The results of the experiments are given in Table XXV.

TABLE XXV.

Current = 5.00 ampères.

Length.	Resistance.
0.50	3.70
1.00	4.42
1.00	4.33
2.00	5.38
2.00	5.38
3.00	7.12*
3.00	7.12*
4.00	7.68

Current = 7.95 ampères.

Length.	Resistance.
0.50	1.69*
1.00	2.45
1.00	2.57
2.00	3.06
2.00	3.21
3.00	3.70
3.00	3.66
4.00	4.28
4.00	4.17
5.00	4.93
5.00	4.80
6.00	5.51
6.00	5.37

Current = 9.86 ampères.

Length.	Resistance.
0.50	1.75
1.00	1.97
1.00	2.09
2.00	2.42
2.00	2.42
3.00	2.74
3.00	2.92
4.00	3.32
4.00	3.14
5.00	3.86

These results, when plotted as before, give straight lines, represented by the following equations.

TABLE XXVI.

Current.	Number.	Equation.	E. M. F. $= a \times C$.
5.00	31	$r = 3.30 + 1.09\, l$	16.50
7.95	32	$r = 2.00 + 0.57\, l$	15.90
9.86	33	$r = 1.70 + 0.38\, l$	16.76
			Mean, 16.39

* These values deviate greatly from the curves.

These lines are all less inclined than those for the same current with whistling arc under normal pressure; showing that as the arc is lengthened the resistance proper increases less rapidly when the pressure is reduced. The total equivalent resistance is, of course, greatly diminished by reduction of pressure, but this reduction seems to be wholly due to diminished conductive resistance, the inverse electromotive force for the arcs used, all of which were whistling ones, being even slightly greater than at normal pressure, so far as can be judged from the data at present obtained.

The scanty data at hand would not justify us, however, in concluding this apparent increase to be real. If such proves to be the case, the action is probably of the same nature as that supposed by Edlund to exist in the discharge of electricity in ordinary vacuum tubes; that is, an increase in the opposition offered to the passage of electricity from the solid electrodes, as the pressure of the intervening gaseous medium is diminished. Further experiment, extending to higher vacua, and also with increased density, is necessary before any definite conclusions can be drawn as to this point.

In addition to those facts which are commonly recognized, the following conclusions seem to be justified by our experiments.

1. There is a definite inverse electromotive force for the whistling arc, whose value is approximately 15 volts.

2. The inverse electromotive force for both the silent and whistling arcs diminishes slowly as the current increases.

3. The inverse electromotive force, at least for the whistling arc, is less for the inverted than for the upright arc.

4. The great change in equivalent resistance which occurs when volatile salts are introduced into the arc is chiefly due to a large fall in the inverse electromotive force, although there is at the same time a marked diminution in the conductive resistance.

5. The diminished total resistance of the arc in rarefied air is due solely to a diminution in the conductive resistance.

6. There is some evidence to show that, with considerable reduction of pressure, there is a slight increase in the inverse electromotive force.

Rogers Laboratory of Physics,
June, 1886.